Using image symmetries to uniquely align aspheric mirrors to a focus and axis

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ABSTRACT

The Point Source Microscope (PSM) is used to find five aberrations related to the symmetries of the autostigmatic image viewed when aligning aspheric mirrors to a point along an axis. These five aberrations exactly match in number the five degrees of mechanical freedom required to align the mirror to an axis and thus provide an exact solution to a unique focus and alignment to an axis. We show how the PSM is used to capture and analyze a set of images as the PSM is moved through focus using the symmetry properties of the image.

Keywords: Optical alignment**,** image symmetry, low order aberrations, autostigmatic or Point Source Microscope

1. INTRODUCTION

When testing an aspheric mirror for figure quality during polishing, the most frequently used test is to place an interferometer at the mirror focus and autocollimate the light from a return flat or sphere. To keep the example simple, I will assume the mirror is a parabola tested against a plane mirror, but the parabola could as easily be any stigmatic optical system tested against an auto-reflecting mirror. Further, while most people are familiar with doing the test with an interferometer, I will also talk about doing the test with an autostigmatic microscope (ASM)[1,2] using a Star test[3] to show the similarity of the two approaches.

When testing the mirror for figure error, whether to find high regions for further polishing or for final acceptance, the mirror must be as perfectly aligned in the test setup as possible to avoid polishing in an error that is due to misalignment. It is this aspect of alignment that is the subject of this paper, the avoidance of misalignment during the polishing or final acceptance test of quality. Since it is impossible to achieve a "perfect" parabolic surface on the mirror, the alignment must also be the best possible alignment given the slight imperfection of the surface from the true, desired surface.

To begin we will start with an over simplistic example of testing a spherical mirror to assess its figure quality and show how misalignment can have an effect in even this simplest of cases. The concepts of this example are then extended to the case of the parabola and its alignment. This leads into the idea of stigmatic, or Star, image symmetries and their usefulness in alignment. We end by suggesting possible further methods of alignment.

2. SPHERICAL MIRROR TESTING

One might ask why such a simple example as testing a sphere should even be considered since nearly everyone is familiar with this test. The answer is that it is not as simple as it first seems, particularly in this age of trying to make "perfect" optics for photolithographic systems, for example. In the days before interferometers, surface figure, or irregularity, was measured using test plates, or glasses, of an equal but opposite radius to get interference fringes. To see the bend in the fringes to get a feel for the irregularity it was typical practice to put enough wedge between the two surfaces to get 10 or so interference fringes. Then you could lay down a straightedge to judge the lack of straightness in terms of the average fringe spacing. If there were no wedge or tilt in the fringe pattern the most you could tell was the difference in power, or radius, between the surfaces.

This desire to see a few fringes in the interferogram held over in the days of the first interferometers. In particular, the earliest interferometers could do nothing but show the fringe pattern the same way as a test plate, but the interferometer gave a way of getting the fringes without having to contact the surface being tested. The focus of the interferometer or ASM is placed at the center of curvature of the spherical surface so light from the focus strikes the surface at normal incidence and reflects back to the focus. The focus, a point in space defined by 3 degrees of freedom, is *all* that is necessary to locate a spherical surface in space.

To this day some opticians still like to see the fringe pattern of straight fringes as a double check on whether the interferometer is giving them believable data via the contour map it produces. There is a problem, however, with having tilt between the spherical surface under test and the reference, or transmission, sphere of the interferometer; the tilt introduces astigmatism that is not in the surface but is due to the tilt. In almost all cases the astigmatism is too little to worry about but if the surface is very fast then there can be a problem. As the subject will come up in the next section, a consequence of having tilt between the test instrument and surface being tested is that the rays from the test instrument are not incident normal to the spherical surface being tested. The lack of normality is small but is the underlying reason producing the aberration due to misalignment. This effect is also known as retrace error[4].

A second problem is that transmission spheres are optimized for use on axis and can themselves introduce aberrations if used off-axis, that is, with tilt between the reference sphere and the spherical surface under test.

The point I am trying to make is that when you use an interferometer you first want to make sure the transmission sphere is aligned to the interferometer optics themselves and then when taking an interferogram of the surface under test, that there is as little tilt as possible between the surface under test and the reference sphere. With phase shifting methods now available on all interferometers there is no need to have fringes visible in the interferogram and having a single fringe "fluffed out" over the aperture leads to the least error due to alignment between the surface under test and interferometer, and least error due to the interferometer itself.

The same logic applies to using an autostigmatic microscope, you want the reflected image to return over the same path as the light left the microscope. To do this, you first find where the light is leaving the microscope by focusing on a specular surface to obtain a Cat's eye image and then setting the crosshairs in the eyepiece or detector on the Cat's eye image. This step is completely analogous to aligning the reference sphere in the interferometer to the interferometer optics. Another way of thinking of this operation is picturing the boresighting of a riflescope. You want the crosshair in the eyepiece coincident with the bullet hole in the target.

Once the "boresighting" is finished with either the interferometer or autostigmatic microscope, if the light reflected back into either instrument shows tilt in the case of the interferometer, or is not centered on the crosshairs, the test instrument is not centered, read "aligned", with the optic under test. If it is not centered there will be an error introduced in the test result at some level. The error introduced will be larger for large NA cones of reflected light. Whether an alignment error is of concern is easily found by analyzing the test with a lens design program. In all but the most precise cases the error will not be a concern, at least for spherical surfaces. Matters get worse for aspheric surfaces.

3. TESTING A PARABOLA IN AUTOCOLLIMATION

This example is just one step more complex than testing a sphere, but there is a fundamental difference, the parabola has an axis where the sphere has none. The parabola is defined in space by 5 degrees of freedom, 3 translational DOFs related to the spherical vertex radius of curvature, and 2 related to the location of the vertex relative to the center of curvature and focus. For a very slow parabola you can still test the mirror as though it was a sphere and directly measure the aspheric departure from a sphere. (There are cases where a sphere will work as well as a parabola as a collimator for testing other optical systems if the sphere has a long enough radius that the departure between sphere and parabola is small enough to be inconsequential.) This case, however, is not of interest other than the same concerns about being precisely at the center of curvature for the truest measure of figure error. It should also be added that a faster parabola could be tested at it center of curvature provided a computer generated hologram were used to turn the spheric wavefront into a spherical one. Alignment is critical in this case but is not the subject of this paper.

Generally, a parabola would be tested against a plane mirror in autocollimation with the focus of the test instrument at the parabola focus since light from the focus is projected toward infinity from the parabola as shown in Fig. 1 where the parabola is shown with a central hole as this is how almost all symmetric parabolas are used. Because the vertex of the parabola is missing due to the hole there is no convenient method of locating the vertex to establish the axis and the 3 degrees of freedom at the focus are not enough. However, the first step of alignment is the same as for the sphere, the light from the focus of the test instrument must return after reflecting from the flat and two bounces off the

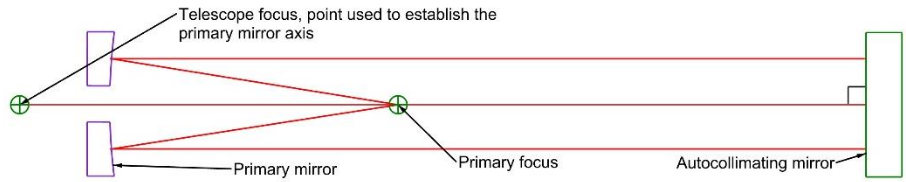

Figure 1. Autocollimation test for a symmetric parabola

parabola to the focus. Which components are adjusted to achieve this are immaterial, the goal is to get the focused light back with no tilt or centered in the crosshairs because as we have seen this means the light is incident and reflected at normal incidence from the flat.

The reflected wavefront, or image in the ASM, will be aberrated but it is easy to adjust for minimum tilt and focus in the interferometer. Similarly, it is easy to get a well centroided and focused spot in the ASM. To remove the aberrations which are coma in two directions, we must adjust the flat normal to the axis of the parabola. There are two ways of doing this, a very deterministic way by making sure the focus and vertex center of curvature lie on a normal to the flat or use aberration minimization. We will discuss the use aberrations since that is the motivation of this paper.

4. REDUCING THE ABERRATIONS

Just as with getting the reflected spot back centered in the ASM, we use any adjustment to reduce the aberrations with the constraint that a compensating adjustment must be made to keep the reflected spot centered, or in the interferometer, the interferogram free of tilt. Obviously, if you go the wrong way while making the adjustments the aberration will get worse, so this is a quick way of knowing you are going in the right direction.

At first glance this aberration reduction seems completely arbitrary, but it is not. What you are doing to reduce the coma is to rotate the parabola around its center of curvature until it axis is normal to the flat. This can be accomplished in several ways, but this is what any of the methods is doing. When the parabola axis is normal to the flat there will be no coma. The question is then what is good enough and this depends on the particular specifications for the system at hand.

If you are using an interferometer for alignment, you immediately get an rms number for the wavefront error. Remember the number is twice as large as the actual wavefront error because of the double bounce off the parabola. If you are using an ASM, we only have the image symmetry to work with and this is what is described next.

Before we proceed there, one might ask why consider using an ASM to do the alignment. The answer is ease of use. An ASM is lightweight and small meaning that it is easily mounted on an xyz stage for easy positioning at the parabola focus. The ASM is easy to move as the aberrations are reduced because in making adjustments it is usually easier to move the test instrument than the parabola, and easier still if the test instrument is small. Moving a typical interferometer weighing over 20 kg and measuring about a half meter square is not easy. In fact, even if the result has to be an interferometric test, if the parabola test is first aligned as well as possible using an ASM, then when the interferometer is moved into place it only has to be moved precisely in 3 degrees of freedom, so its focus is where the ASM focus was. The angular alignment is non-critical if the interferometer covers the full aperture of the parabola.

Another reason for using an ASM is that it can be used at any visible wavelength and into the near UV and IR. For the parabola this would make no difference but could be a requirement for refractive systems. The ASM also uses partially coherent light so most unwanted coherent reflections are eliminated.

5. USING SYMMETRIES

When an ASM is used to analyze an image of a point source of light that has propagated through an optical system such as in the alignment of the parabola cited above, the near perfect point source of the ASM will be aberrated by the misaligned test. We show that if matrices of pixel intensity data are taken at discrete steps going through focus, "best focus" may be repeatably found to an order of magnitude better that a visual assessment of best focus. In addition, if the image is analyzed in terms of 4 spatial symmetry groups in the plane perpendicular to the focus direction, quantitative measures of pseudo "aberrations" can be obtained that are analogs of 0 and 45 degree astigmatism and 0 and 90 degree coma. Good alignment of optical systems depends on driving these aberrations as close to zero as possible. Having quantitative measures of the aberrations provides information for doing so in a fast, efficient, and deterministic manner.

5.1 Explanation of the method: A set of images from an ASM, the Point Source Microscope[6] in this case, were collected at the back focus of a poorly centered lens so the images contained some aberrations. A total of 7 8-bit images were obtained spaced every 5 μm about what looked visually like best focus. The exposure level was set so there were no saturated pixels.

The 1024 x 1280 pixel files were reduced to 21 x 21 pixels roughly centered on the image. The upper left pixel of each smaller matrix was the same pixel location in the full matrices for all 7 smaller matrices. Because the PSM centroids on the pixels above threshold, typically set at about 140 out of 255 bits, the x, y centroids for the 7 focus positions were

Axial position (um)	x (pixel number)	y (pixel number)
-15	below threshold	below threshold
-10	below threshold	below threshold
-5	12	10.5
0	11.5	10.5
5	11.5	10.5
10	11.5	10.5
15	below threshold	below threshold

showing that the center of gravity of the image pixels above threshold did not change by more than 0.5 pixels in the 15 μm range where there were pixels above threshold. When measured in the object space of the PSM using a 10x objective, each pixel is slightly less than 1 μm in size in object space.

The reason for concern about the centroid of the image is that in the analysis done on the 21x21 grid of intensity data, the data were rotated and flipped about the central pixel, that is, pixel 11, 11. To do the analysis entirely correctly the intensity data should be shifted so the centroid determined by the "center of gravity" is at the "4 corners" intersection of a square grid of data with an even number of rows and columns. The even-odd and odd-even symmetry groups are quite sensitive to the point about which the data are flipped. In the data below, the centroiding is good only to 0.5 pixels but this is good enough to demonstrate the method. In future work the images will be centroided more precisely.

Since the reason for quantitatively determining the "aberrations" is to know when an optical system is aligned as well as possible, it is important that the centration of the image does not influence the scale of the aberrations derived from the analysis. Exactly how to do the analysis best is yet to be determined. We are presently showing how one implementation of the method works.

5.2.1 Raw data used: The seven 21 x 21 matrices of pixel intensity data were divided by 247, the highest 8 bit intensity value in any matrix, to normalized the data. The normalized intensity maps below show the raw data.

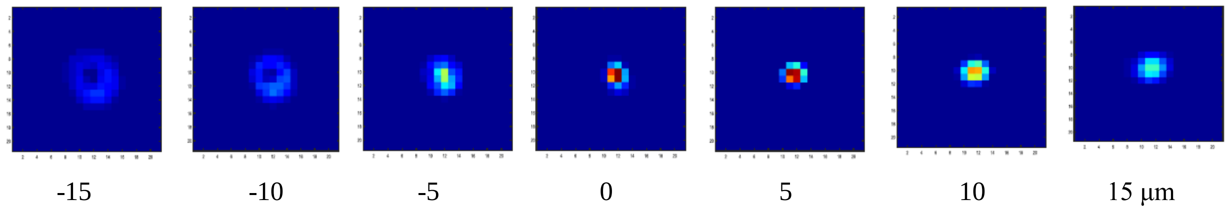

Figure 2. Spacing of raw data going through focus. The 0 does not indicate best focus but an arbitrary value.
(In the normalized data, the dark blue is 0 intensity and the deep red is 1 in chromatic order)

It is clear that the range of raw data is not symmetric about what appeared to be best focus, at 0 of the scan, but was gathered on a visual estimate of best focus. The collapse of the lower intensity spot into a more concenrated one and then drop in intensity is clear from the data. It is also obvious that the image is not symmetric about its center.

5.2.2 Obtaining the symmetric portion of the image: To find the symmetric part of the image the raw data at each focus position was rotated 90 degrees 3 times about pixel 11, 11, and the original and three rotated image matrices were averaged. The resulting symmetric images are below to the same scale.

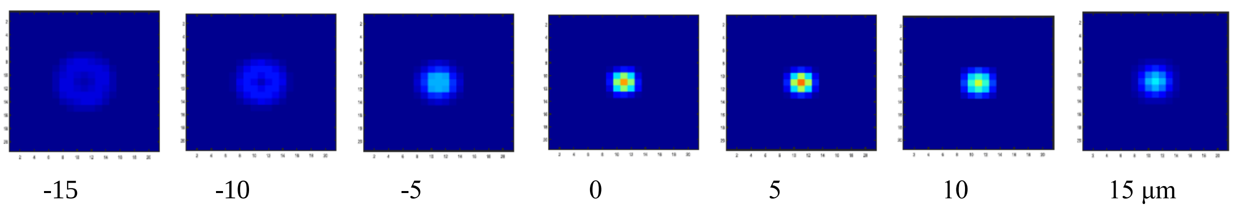

Figure 3. Symmetric portions of the image obtained by rotating the raw date and averaging
The numbers below the images are their positions through focus when the data was taken

5.2.3 Obtaining the asymmetric portion of the image: The matrices of these symmetric portions of the image (Fig. 3) were subtracted from the raw image data (Fig. 2) to give the residual asymmetric image data. This operation leads to some intensity values being negative, so the color maps were renormalized from -.45 to .67 to give a better feel for the asymmetric parts. The blue-green background around the images corresponds to zero.

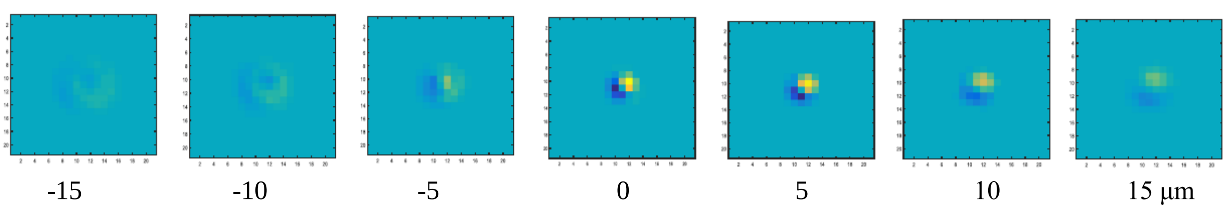

Figure 4. The asymmetric portion of the image at even, 5 µm distances through focus
The blue-green color around the images is zero in the renormalized images

In a Cartesian coordinate system a function in the x, y plane, F(x, y), can be represented by 4 symmetry groups, F_{ee}, F_{oo}, F_{eo} and F_{oe} where e stands for even and o for odd. The components of these symmetry groups are[7]

$$F_{ee}(x, y) = (F(x, y) + F(-x, y) + F(x, -y) + F(-x, -y))/4$$
$$F_{oo}(x, y) = (F(x, y) - F(-x, y) - F(x, -y) + F(-x, -y))/4$$
$$F_{eo}(x, y) = (F(x, y) + F(-x, y) - F(x, -y) - F(-x, -y))/4$$
$$F_{oe}(x, y) = (F(x, y) - F(-x, y) + F(x, -y) - F(-x, -y))/4$$

We use these 4 equations to find the 4 symmetries of the asymmetric part of the images. These symmetry components correspond to Matlab functions for flipping matrices. F(-x, y) is fliplr(F(x, y)), F(x, -y) is flipud(F(x,y) and F(-x, -y) is fliplr(flipud(F(x, y)). Other than dividing by 4 these functions are nothing more than relabeling the matrix cells so the operations are very fast and almost free of any computation. Below are the asymmetric portions in F_{ee}, F_{oo}, F_{eo} and F_{oe} in order through focus.

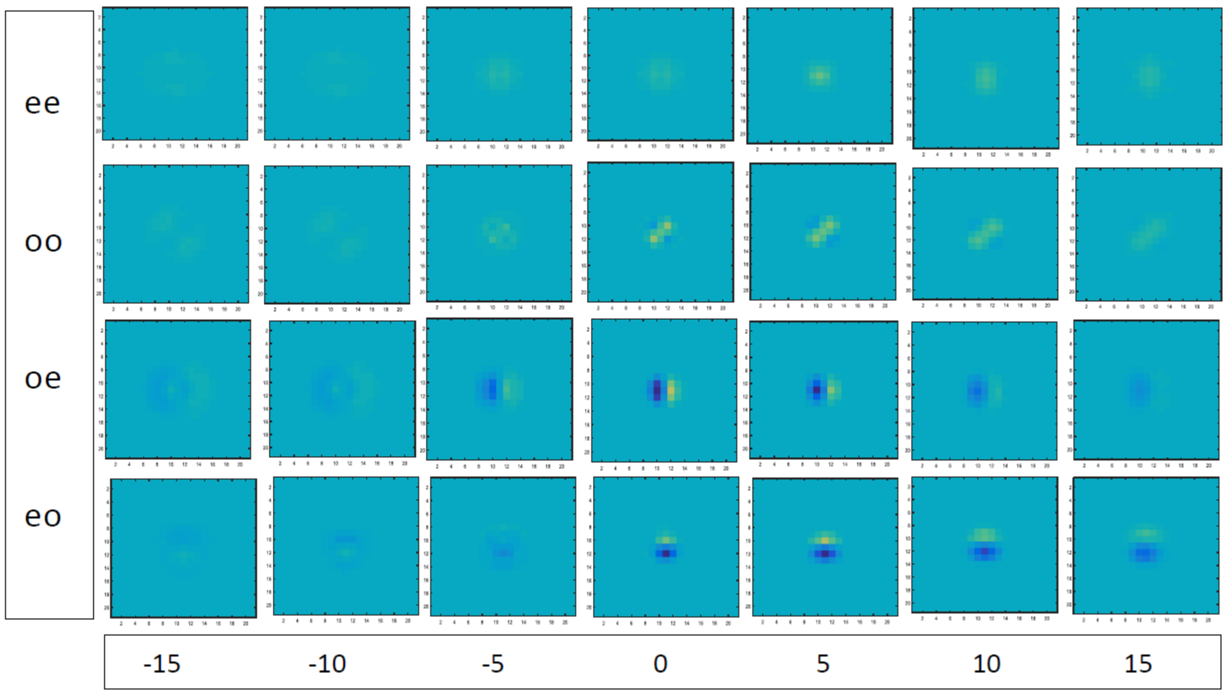

Figure 5. Asymmetric portions of the of the image going through focus (Units are μm)

The residual, or asymmetric, parts of the image were renormalized, or scaled, from -1 to 1 since once the symmetric part of the image was subtracted from the raw image, the intensity maps have negative values. The images in Fig. 4 (where 0 is blue-green) are the entire asymmetric part of the image as the PSM was stepped through focus in 5 um steps. Then we used the symmetry operators to give the next 4 sets of images in Fig. 5 that are the 4 symmetry groups of the asymmetric part of the image through focus. It is clear why the symmetric part of the image is removed as the first operation because it also has ee symmetry.

An examination of the Fig. 5 shows that the ee group of the asymmetric images looks like 0 degree astigmatism and the oo group looks like 45 degree astigmatism. The oe group looks like x coma and the eo group like y coma. For all the images best focus appears to be between focus positions 0 and 5 um with best focus being about 4 um.

In fact, this method of using image symmetries effectively determines all the 3rd order aberrations due to misalignment and high order aberrations of the same symmetries. Since we require coincidence of the return spot with the outgoing spot there is no tilt in the return wavefront. The symmetric part of the image accounts for focus and all orders of spherical aberration. The asymmetric terms account for all aberrations having the symmetries of the two orientations of astigmatism and for the two orientations of coma. Since 3rd order aberrations are always larger than higher order aberrations and these are minimized, using the symmetry groups to maximize the symmetric aberrations and minimize the asymmetric gives an optimum alignment.

5.3 Quantifying the symmetry groups: To give a quantitative feel for the coefficient value of each of the images in the sense of a Zernike coefficient, for example, the pixel intensity matrix elements were squared individually and then summed to give the value. These coefficient values are shown in the graph where **sumsq** is the value of the original image, **symsq** is the value of the symmetric part of the image, **ressq** is the asymmetric part and the final 4 the asymmetric components. All values peak at about 4 um except the eo and oe parts. The value in these components probably depends on the specific definition of the center of the matrix as noted above in 5.1. With a more precise centering of the matrix before rotation these values would probably also match the other cases.

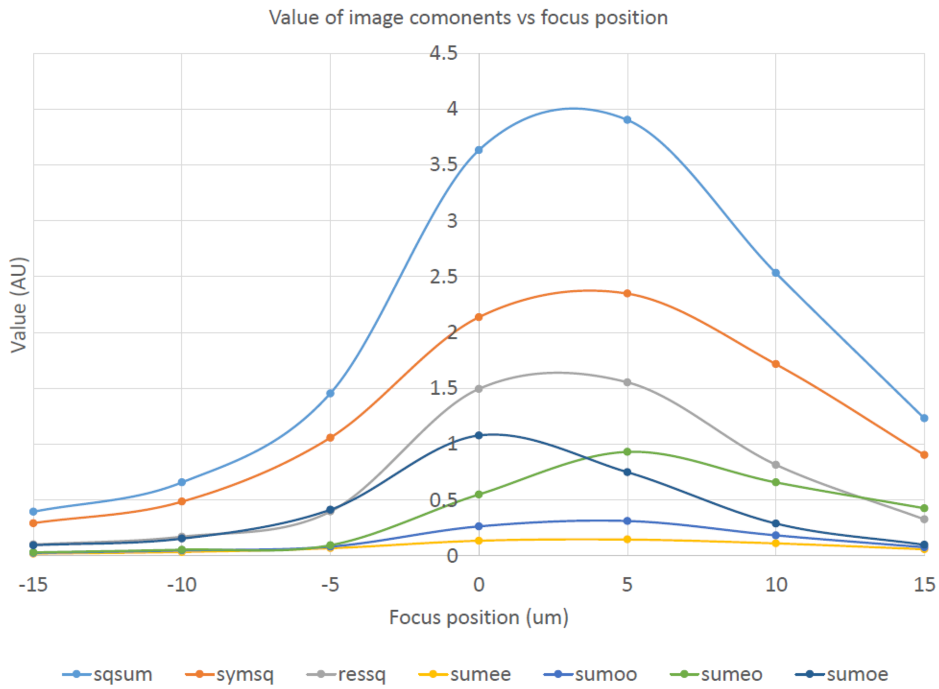

Figure 6. Image symmetry component values (coefficients) through focus

6. USE OF IMAGE SYMMETRY FOR ALIGNMENT

While an optimum method of quantifying the 5 "pseudo" aberration values has not been determined, it is easy to obtain these aberration values from the Star, or stigmatic, image data, and the method of finding the values is computationally fast. It is expected that using a motorized stage, or PZT microscope objective focusing stage, the data needed to analyze the image can be obtained rapidly so that real time so that quantitative data is available for alignment. This means that improvements in alignment adjustments are made with real time feedback. Further, the 5 aberrations based on image symmetry are just enough degrees of freedom to align almost any optical test or system since there are only 5 independent degrees of freedom available for alignment for any optical component.

6.1 General procedure for alignment: The general method of alignment in autocollimation is best shown in a diagram. Fig. 7 shows the steps for alignment where the first step (top) defines the axis of the test as the line between the PSM focus and the normal to the autocollimating plane mirror. The point focus and the two angles defined by the mirror completely define 5 degrees of freedom of the axis (4) and a point (1) on the axis.

When a lens, or any optical system (including those with aspheres), is inserted between the focus and flat, and adjusted axially to produce a focused spot, the spot will not, unless you are very lucky, lie on the axis of the test as shown in the second step. Typically, the lens is both decentered and tilted relative to the axis. The focused spot does not return to the focus of the PSM but can be made to do so by either decentering the lens or by a tilt around an appropriate

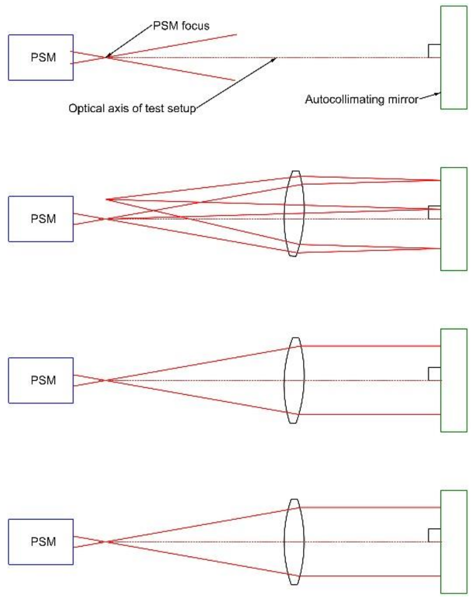

Fig. 7 Four steps of a typical alignment process starting with defining the axis of the process

center of rotation as shown in the third step. When the centroid of the reflected spot lies exactly on the outgoing focus of the microscope objective, the collimated light will strike the flat, or return mirror, at normal incidence. This adjustment to bring the reflected focus coincident with the outgoing focus is equivalent to minimizing the tilt fringes in interferometry to reduced retrace error to a minimum.

Now the centroid of the aberrated spot must be kept centered on the outgoing spot while making compensating adjustments in tilt and decenter of the lens. Clearly a small decenter of the lens will move the reflected spot laterally and this must be compensated by a tilt to keep the spot centered, or the adjustments can be reversed. If the aberrated spot gets worse, you were going in the wrong direction. By making tilts and decenters in both axes while keeping the spot centered and well-focused you are using 5 degrees of freedom; all you have available.

If you are also using the image symmetry to gauge the value of the symmetry components, you have immediate feedback on which adjustments to make and to when you have achieved the best possible alignment given the manufacturing and alignment errors in the optics located between the PSM and the flat. Due to manufacturing errors you may find that the center of the field where all aberrations should be symmetric is not where mechanical dimensions would indicate the center should be. At this point you would have to decide whether to leave an asymmetric image at the center of the field as determined by the mechanics of the instrument or shift to an offset center of the field that gives better on-axis performance.

7.0 CONCLUSIONS

We show a systematic method of aligning a double pass test of an optical system with an interferometer or autostigmatic microscope where the first step is to remove tilt or centroid the return spot on the outgoing focus to eliminate retrace error in the test setup. This alignment ensures the light is incident on the return mirror at normal incidence. The alignment continues by making compensating adjustments to the optical system in tilt and decenter to keep zero tilt or good centration while reducing the aberrations.

We show that five pseudo “aberrations” derived from image symmetry are used to provide feedback to aid in the rapid reduction of aberrations using the five degrees of mechanical freedom available for adjustment. The means of calculating the "aberrations" is fast so there is real time feed back for adjustment to a best case alignment.

8.0 REFERENCES

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