Alignment using axicon plane gratings

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ABSTRACT

Axicon gratings are computer generated holograms of equally spaced concentric circles printed on a plane substrate. When illuminated by a point source of light they create axes in space defined by a line between the point source and the center of the grating pattern. The axis can be viewed in either transmission or reflection with an autostigmatic microscope. The axis created by the grating can be located to less than 1 um in translation and depending on distance from the grating to less than 1 microradian in angle. Several examples of such a use in alignment are explained.

Keywords: Axicon, optical alignment, Bessel beams, autostigmatic microscope, diffraction grating

1. INTRODUCTION

While this paper is about alignment with axicon plane gratings the paper will be difficult to follow without going reviewing the background and history of both axicons and autostigmatic microscopes. First, the history of the axicon will be discussed along with the reason for its invention, optical system alignment. Then it is shown how a plane grating with a very simple pattern behaves like a physical glass conical axicon, but that it is much easier to fabricate an essentially perfect axicon with a grating than make the axicon from a piece of glass or metal.

From there the properties of axicons are discussed to show why they are so useful in optical alignment and to show the difference between illuminating the axicon with a plane or spherical wavefront. Although the performance is nearly the same independent of the method of illumination it is important because it is so much easier (less expensive) to create a near perfect, bright point source of illumination than a large diameter, diffraction limited plane wavefront. Also, most of the literature about axicons assumes the illumination is by a plane wave giving the impression that this is the only way an axicon will work to create an axis.

Along the way we will touch on Bessel beams, contrast a conventional laser beam with the axis created with an axicon grating and show why the axicon beam may be considered more precise when used for alignment. We will also discuss why axicons have not been used very much for alignment purposes in the past given that their use makes so much sense when looking at papers suggesting the use of axicons. Just before talking about alignment itself, we introduce the concept of an autocollimating flat with an axis. No longer does the flat define 3 degrees of freedom but 5 when it has an axicon grating printed on its surface.

From here we discuss the alignment first of refractive optics and then move on to reflective optics. While the discussion is meant to be complete it will be obvious that there are subtleties to the subject that bare further investigation. It should be obvious by the end of the paper that there are still many what ifs.

2. HISTORY OF THE AXICON

In 1954, John McLeod of Eastman Kodak Company wrote a paper in JOSA titled “The Axicon: A New Type of Optical Element” where he said “The word axicon has been coined to cover a type of optics…[that have] the property that a point source on its axis of revolution is imaged to a range of points along its axis”.[1] McLeod credits van Heel[2] with prompting his research into the axicon. Van Heel’s diffraction methods had the precision MacLeod desired but suffered from lack of a sufficiently bright light source to make the methods generally applicable.

Several points are made by McLeod about the glass axicons he had made; first they made possible very precise angular measurement, slightly better than 1 μradian. Second, good axicons were difficult to make because they are extremely aspheric. Finally, reading between the lines, even though his axicons were an improvement on van Heel’s methods, having a brighter light source would have made the axicon more useful.

The final point McLeod makes in his paper will carry over directly to the discussion axicon gratings. When an axicon with a plane back is aluminized on either surface, and an autocollimating[3], or autostigmatic[4], microscope is used to view the reflected image along the axis of the axicon, "…the images will always be at unit magnification for any distance from the cone out to its maximum range. Moreover, the size of the diffraction pattern will always be constant. The illuminance will be inversely proportional to the first power of the distance, not to the usual second power. Because of this the angular diameter of the ring image remains constant…".

In 1960 McLeod wrote a second paper[5], "Axicons and their Uses" that has a diagram which is a precise analog of the diagram we will show when we get to describing the alignment of refractive optics. The upper right of Figure 1 is a redrawn picture of Figure 4 from his paper with a pinhole source and a human observer looking into an eyepiece. In the black and white drawing it is not immediately clear that there are two bundles of light that will refocus back in the autostigmatic microscope, one bundle formed by the rays that hit the plane mirror at normal incidence and another by the rays creating the axis of the axicon. As the plane mirror is tilted away from being normal to the axicon axis the two spots will move away from the center of the eyepiece but at different rates depending on the axicon cone angle and the distance to the plane mirror. This is the same behavior we will see with the axicon grating and a Point Source Microscope[6], a modern version of the autostigmatic microscope.

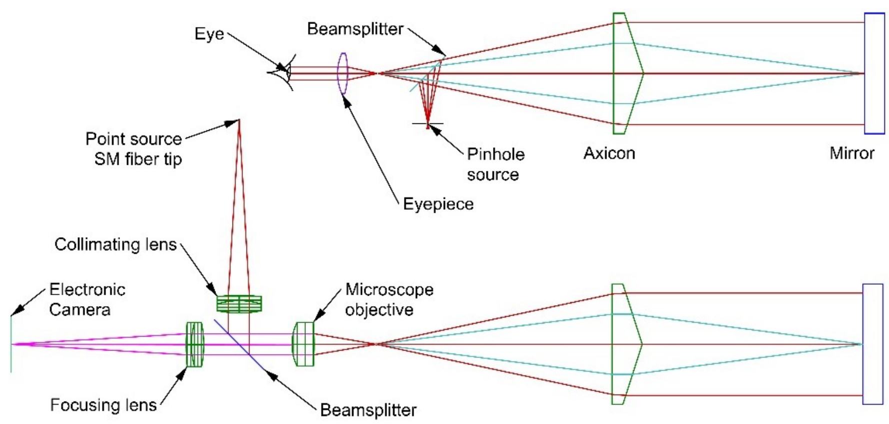


Figure 1. In the upper right is a redrawn version of Figure 4 in from McLeod, Ref. 5. The lower part of the Figure shows the axicon and mirror viewed with an autostigmatic microscope to illustrate that the optics of the two versions are analogues with the modern version having a brighter source and more sensitive detector.

As McLeod described many possible uses of the axicon he concluded that it was hard to say where this technique for alignment would lead but suggested that a machine tool could be guided photo-electrically along the axicon axis beam rather than using steel ways, the usual method of governing machine tool position. In the very last paragraph, however, he said the real problem with this method was "light flux". The same year as McLeod published this paper, Javan, Bennett and Harriott[7] first demonstrated a CW, HeNe laser and light flux would no longer be a problem.

3. AXICON GRATINGS

Now that the historical aspects of axicons have been introduced and how they are useful for alignment we turn to axicon gratings. These grating are nothing more than a set of evenly spaced concentric circles printed on, or etched into, a glass substrate using e-beam lithography with the same precision as photomasks used to make computer chips. In the case of those described in this paper we have used both chrome on fused silica photomask substrates, and phase gratings etched into fused silica. The phase gratings are more efficient for use with refractive optics alignment while the chrome on glass seem better suited for aligning reflective optics in double pass setups as we will show. An example of a 75 mm square chrome on fused silica photomask grating with 20 μm line spacing is shown in Figure 2. The 20 μm line spacing along with the 0.64 μm wavelength illumination gives a grating angle of 0.032 radians or about 2°.

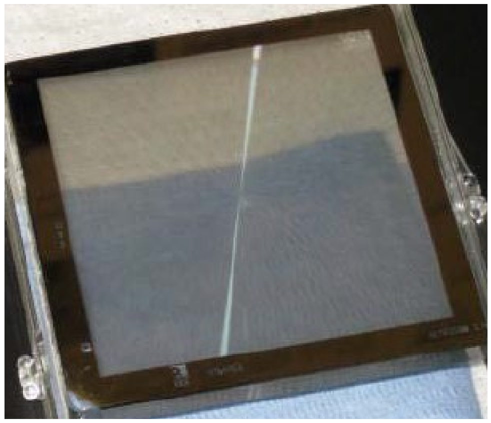


Figure 2. Chrome on fused silica photomask substrate 75 mm square with 20 µm line spacing.

When the axicon grating is illuminated on or near its axis by the point source in an autostigmatic microscope, a Point Source Microscope (PSM) in this case, the reflected image looks like that shown in Figure 3, a bright central spot with increasingly dimmer concentric circles. Figure 3 was cropped to show only the central 20% or so of the ~1 Mp frame.

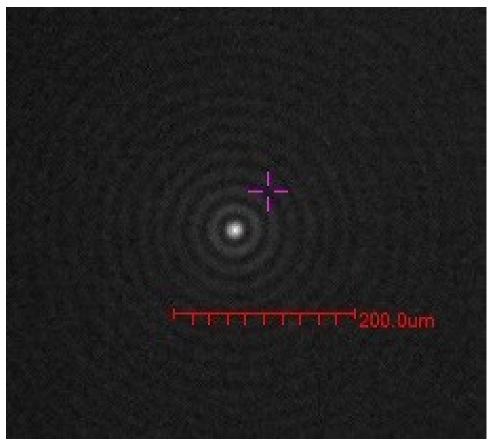


Figure 3. Reflected image from the grating in Figure 2 where the autostigmatic microscope focus was close to the axis of the grating (indicated by the magenta crosshair) and about 200 mm from the grating.

Because it is difficult to judge intensities from a grey scale picture, a trace was made through the central maximum in Figure 3 to make the plot in Figure 4. Here the camera exposure had been adjusted so the recorded intensity was just under the 8 bit linear range to avoid saturation. The full width, half max of the central peak is about 7 pixels and the first ring around the peak has an intensity of about 25% of the peak. The pixels are about 1 µm in object space and the centroid of the peak can be located to a couple tenths of a micrometer. Since the PSM focus was about 200 mm from the grating this amounts to an angular sensitivity of about 1 µradian taking into account doubling on reflection.

While Figures 3 and 4 give a good feel for the reflected intensity perpendicular to the reflected spot, they give no feel to the intensity axially. Most of the literature (excluding astronomical and light trapping uses of axicons and Bessel beams) concerns the intensity when the axicon is illuminated with a plane wave. However, there is a thorough paper by Dong and Pu[8] that calculates the intensity variation along the axis of the grating when illuminated by a point source, and contrasts that with the case for plane wave illumination. Figure 5 is taken from their paper where the red curve is the intensity for plane wave illumination while the blue is for a diverging spherical wavefront.

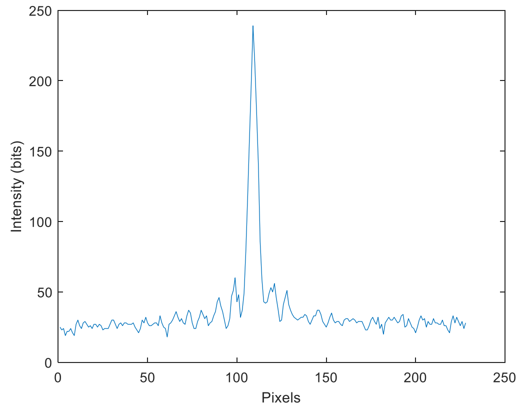
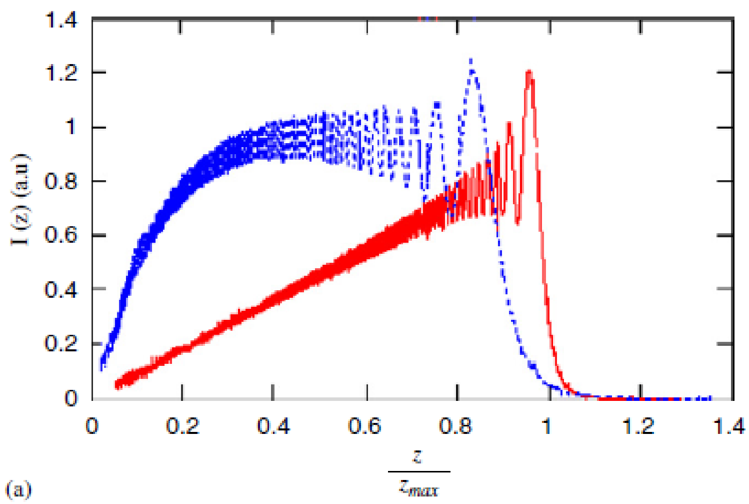


Figure 4. Plot of intensity through the central maximum in Figure 3 to show the width of the reflected image.



(a)

Figure 5. Intensity versus axial distance from an axicon for the axial image in normalized coordinates where the red curve is for plane wave illumination and the blue for a diverging spherical wavefront, that is, point source illumination. (This is Figure 3a from reference 8, Dong and Pu.)

The abscissa in Figure 5 is normalized but for our case the 20 µm line spacing and 75 mm substrate gives the axicon grating a maximum useful radius of about 45 mm. This coupled with plane wave illumination gives $z_{max} = (45*d)/\lambda = (45*20)/0.64 = 1406$ mm for the region where the diffraction free or Bessel beam exists[9,10]. Figure 5 also shows the behavior McLeod observed, that as the image intensity increases it oscillates with distance from the grating until z_{max} is reached whether plane or spherical wave illumination. With spherical illumination the increase is non-linear and reaches a maximum quicker than plane wave illumination. Reference 10 goes into this in more detail and gives a good review of so called Bessel beams and their uses outside the realm of alignment.

4. BESSEL AND LASER BEAM PROPAGATION

Since we are talking about alignment one might reasonably ask why not just use a HeNe laser beam and a quad cell, for example, instead of creating a Bessel beam with an axicon. Even though the laser beam will be larger in diameter than the Bessel beam this really does not matter if one uses a nulling technique such as a quad cell and you do not have the fall off in intensity at z_{max}, the laser beam just keeps going.

From a practical viewpoint, many alignment tasks are most easily implemented in a double pass arrangement and here it is difficult to incorporate the laser though certainly not impossible. A more important concern is with air turbulence. A laser beam travels through a volume of air that is nearly constant in diameter of a few mm. The Bessel beam is formed by light rays a distance from the axis of the grating so they travel through a substantial volume of air until the final focus and the fluctuations in the position of the image are substantially reduced by averaging over the volume compared with transverse fluctuations in a collimated laser beam. This robustness to air turbulence is also borne out by recent research.[11]

5. AXES CREATED BY THE AXICON GRATING

When the axicon grating (AG) is illuminated with a point source of quasi-monochromatic light centered on the grating pattern, the light transmitted through the grating will form a line of bright spots beyond the grating defined by the location of the point source and the center of the grating pattern, see Figure 6. The bright spot varies in intensity as the viewing point is moved along the line, or axis, produced by the spot as was seen in Figure 5. The bright the spot is surrounded by uniformly spaced concentric rings whose intensity falls off with distance from the central bright spot as was seen in Figures 3 and 4. If the point source is moved too far off the axis of the grating the central spot becomes a more complex pattern[12] but in most practical cases of alignment this is never a problem.

The AG may also be used in reflection where two axes are formed, each axis defined by the point source of illumination and the center of the pattern of circles on the grating as seen in Figure 7. One axis is transmitted through the grating as previously described and is a continuation of the line defined by the illuminating point source and the center of the pattern. The other axis is a mirror image of the first axis in the plane containing the first axis, that is, the second axis is mirrored about the center on the grating pattern. The second axis starts at the center of the pattern and reflects back in the direction of the illuminating point source.

A final word about the axicon grating before proceeding on to its use for alignment. Since the grating can be printed on a photomask substrate with a flatness of λ/4 or better, the axicon grating can be thought of as an autocollimating flat with an axis. This means that the grating not only defines 2 angular degrees of freedom and a distance along its axis, but 2 additional degrees of freedom perpendicular to its axis. For rotationally symmetric optical systems this completely defines all the necessary degrees of freedom for alignment.

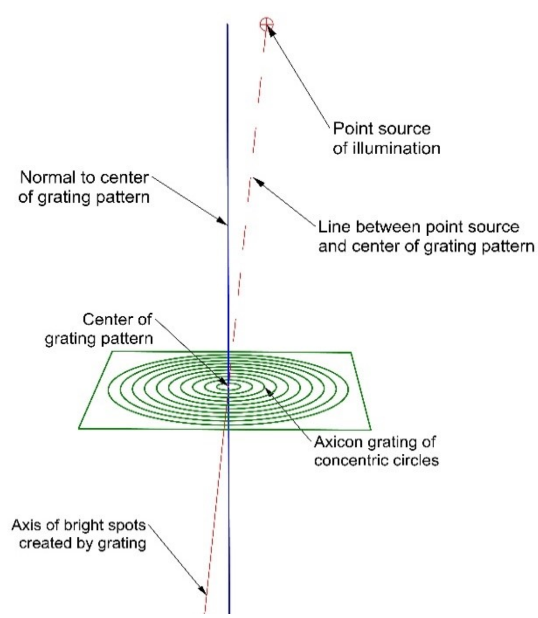


Figure. 6. Axicon grating with point source displaced from the line normal to and through the center of the pattern. The axis of bright spots formed lies beyond the grating on the line joining the point source and pattern center.

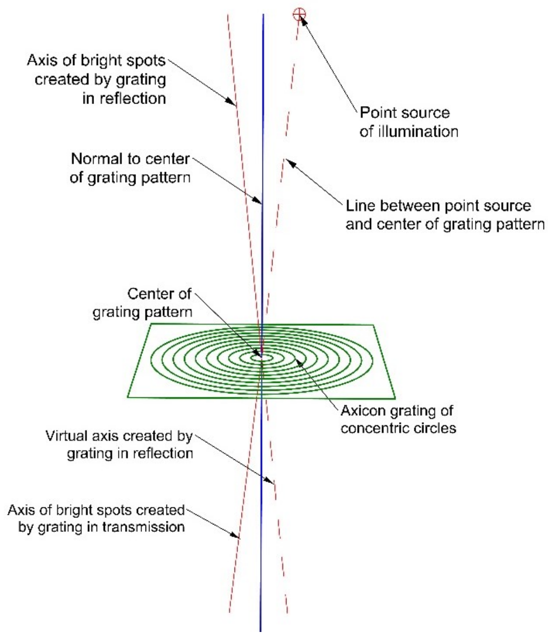


Figure 7. Axicon grating used in reflection with the point source displaced from the line normal to and through the center of the pattern. The axis of bright spots formed in reflection lies on a mirror image of the line joining the point source and pattern center.

6. BACKGROUND ON AUTOSTIGMATIC MICROSCOPES

Since the experimental work here was carried out using a contemporary autostigmatic microscope, the Point Source Microscope (PSM), it is worthwhile looking at how the optics of the microscope work for a full understanding of performing alignment with an autostigmatic microscope (ASM). An autostigmatic microscope[4] has a point source of light directed at an objective lens through a beamsplitter such that the objective lens produces an image of the point source and the wavefront from that image continues on expanding in space as a nearly perfect spherical wavefront.

If a specular surface is placed at the focus of the objective, light is reflected back into the ASM and forms an image (a Cat's eye image) in the object plane of an eyepiece or on an electronic detector. The reflected spot image is precisely conjugate laterally with the point source creating the image so that if the crosshair in the eyepiece is adjusted to be coincident with the reflected image, the crosshair shows the precise lateral position of the focused spot exiting the objective as shown in Figure 8. In other words, when looking into the eyepiece, the center of the crosshair is precisely conjugate with the objective focus and, consequently, the point source creating the focused spot. In an ASM with an electronic detector, electronic crosshairs are centroided on the Cat's eye image on the detector. When the crosshairs are centered on the Cat's eye image, the outgoing focus is precisely conjugate with the point source and the crosshairs in the eyepiece or on the detector.

Now if the focus of the objective of the ASM is placed near the center of curvature of a concave sphere, the reflected light will form a spot on the detector that, in general, will be shifted from the center of the crosshairs as shown in Figure 9. The distance of the reflected spot from the crosshairs will be exactly twice the distance the ASM focus is shifted laterally (decentered) from the center of curvature of the sphere times the magnification of the ASM. Since the radius of the sphere is known, or can easily be measured, the distance between the reflected spot and the crosshairs also gives the angular error, or tilt of the spherical surface, relative to a line between the ASM focus and center of the aperture of the sphere.

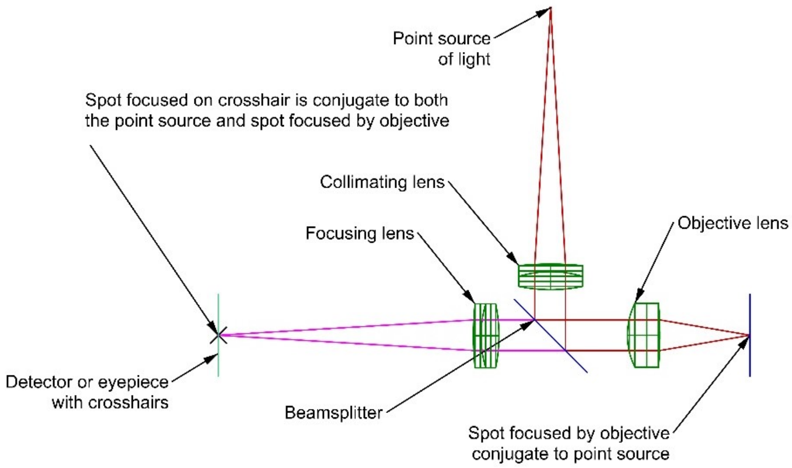


Figure 8. An autostigmatic microscope (ASM) focused on a specular surface to produce a Cat's eye image on the detector conjugate to the illuminating point source as a means of positioning reference crosshairs.

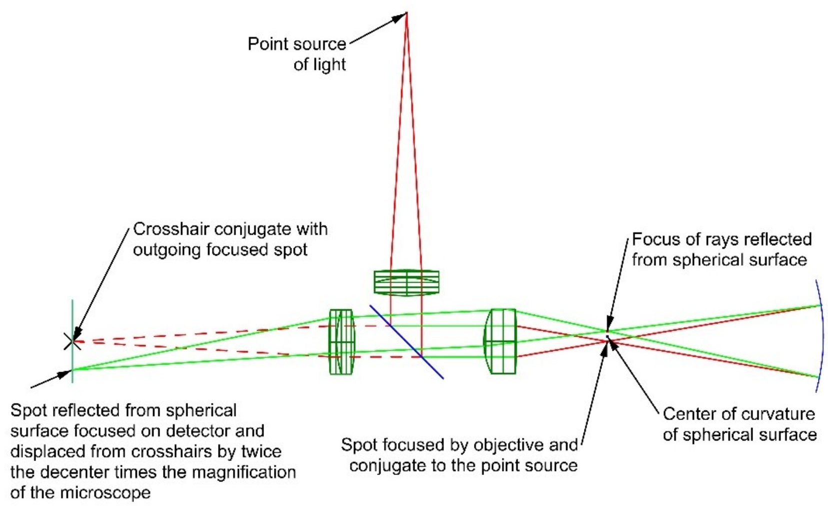


Figure 9. Autostigmatic microscope near the center of curvature of a concave spherical surface. The reflected focus is a distance of twice the decenter of the center of curvature from the out-going focused spot and falls on the detector at a distance from the crosshairs of twice the decenter times the ASM magnification

7. CENTERING WITH THE AXICON GRATING

With this background on both the AG and the ASM, consider centering a meniscus lens using the two. Assume the meniscus will be placed between the ASM and the AG with the concave surface facing the ASM. The ASM is set axially so that its height above the concave surface is such that the focus of the ASM objective is at the center of curvature of the concave surface. First, the ASM is centered laterally so the reflected spot from the AG is centered on the crosshairs which have previously been centered on the Cat's eye image. This assures the focus of the ASM lies on the axis of the AG because the illuminating spot of light is coincident with the reflected spot.

Now the meniscus lens is introduced and two reflected spots will be visible in the detector, one from the center of curvature of the concave surface and the other from light transmitted through the lens, reflected by the grating and transmitted back through the lens to the objective and detector as shown in Figure 10. Now the lens is decentered and tilted until both spots are again centered on the crosshairs. This means the center of curvature of the lens lies on the axis of the grating and the both surfaces of the lens are normal to the axis of the AG, that is, there is neither decenter nor tilt of the lens relative to the AG. Since both spots are centered on the crosshairs the lens is aligned such that the light transmitted through the lens is undeviated so the alignment relative to the axis of the AG is the same as before the lens was introduced.

It is clear that once the lens is centered the axis reflected from the grating will be in exactly the same place it was before the lens was introduced. This means that if subsequent lens elements are to be aligned to the first element, the procedure is repeated by first moving the ASM axially to the center of curvature of the next concave surface or the back focus of the assembly, and then realigning the ASM laterally until the reflected spot from the grating is centered on the crosshairs. (It is assumed the axial stage is not perfect so the ASM will not be perfectly aligned to the axis of the grating once the ASM is moved axially.) Now the next element can be introduced and aligned in tilt and decenter just as the first one was.

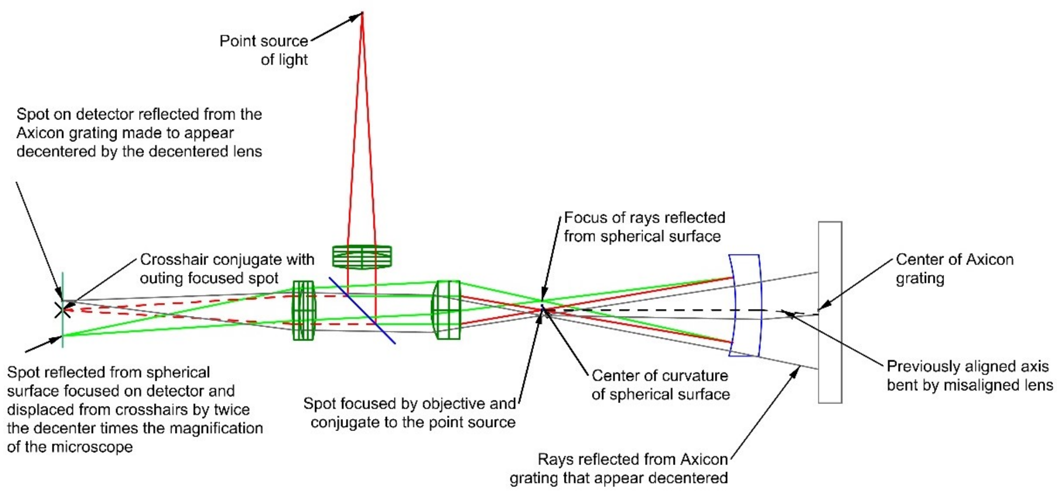


Fig. 5 Decentered lens inserted between the AM and the AG. Two focused spots are seen on the detector, one from the center of curvature of the lens and one from the reflection from the AG as seen through the lens.

8. ADVANTAGE OF A SECOND POINT LIGHT SOURCE

The procedure outlined to this point works perfectly in theory but not well in practice because the light reflected from the AG is less intense than the light reflected from a conjugate of the assembly such as a center of curvature or back focus. This means that with the single source in the ASM the brightness of the two spots on the detector will not be optimum. One or the other spots will be too dim to be useful relative to the other, in general. It is much better to have a second point source of light to illuminate the AG that has an adjustable intensity.

As discussed earlier, the AG works both in reflection and transmission. By placing a SM optical fiber source on the far side of the AG, an axis is created that can be viewed in the ASM. As shown earlier, the axis created this way is the line between the source and the center of the axicon pattern. The ASM is used to assure the fiber point source is located precisely on the axis of the grating. This is done by first aligning the ASM laterally to the AG with the source behind the grating turned off. When the ASM is aligned to the grating axis, the reflected spot will be centered on the ASM crosshairs.

Now the spot created by the source behind the grating must also be centered on the crosshairs by moving the source laterally until the spot it creates also lies on the crosshairs of the ASM as shown in Figure 11. (In practice it may be easier to move the grating and the ASM relative to the source behind the grating until both spots are centered on the crosshairs.)

The second source also helps with this alignment because the PSM electronically centroids on the spots on the detector. By being able to operate the two light sources separately one source at a time can be centered on the crosshairs using the electronic centroid position feedback. This is a more precise way of aligning the two spots because if the spots are coincident, or nearly coincident, it is impossible to improve on the alignment because it is not obvious which spot is being viewed. By turning on one spot, then the other, it is clear how close each spot centroid is to the center of the crosshairs.

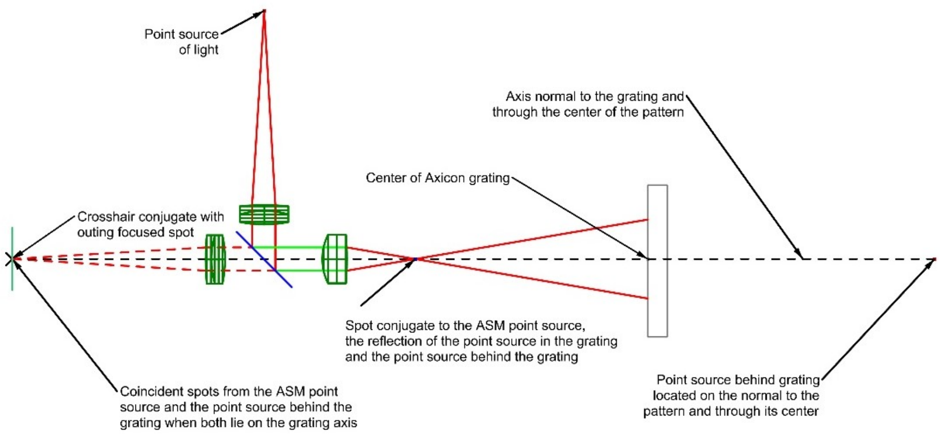


Figure 11. Centering apparatus with two light sources, one in the ASM and one behind the AG. The optical assembly to be aligned will be placed between the ASM and the AG.

9. ALIGNMENT OF REFLECTIVE OPTICS

To align reflective optics we will assume the alignment is to be done in a double pass mode, that the AG is mounted in a tip/tilt stage and that the AG will be used as the autocollimating flat. We will also assume the test will be done horizontally on an optical table or surface plate, and that a rail or straightedge can be clamped to the table. First, the PSM is mounted to an x-y-z stage so that the focus of the objective is approximately height of the center of the AG. Then by observing the reflected axis spot from the AG as the PSM is moved away from the grating along the straightedge, the tilt of the grating about an axis parallel to the table can be adjusted so the axis of the grating remains a constant height above the table. Simultaneously, the tilt of the AG about an angle perpendicular to the table is adjusted so the axis of the grating remains parallel to the straightedge.

Once the PSM is centered on the axis of the AG, the source behind the AG is adjusted so that it also lies on the crosshairs of the PSM. This completes the alignment of the AG and its source so the projected axis of the AG is parallel to the table top and straightedge.

Assume a Cassegrain telescope is to be aligned against the grating with a parabolic primary with a vertex radius of 64.0 mm and an efl of 47.876 mm. The secondary has a vertex radius of 43.45 mm and a conic constant of -6.654. Figure 12 shows the telescope with the mirror spacings and the AG with its source.

Although Figure 12 shows the mirror spacings, the only critical dimension in the setup is the 39.642 inches, the spacing between the primary focus and the system focus. No other dimension matters because the primary focus, along with the autocollimating flat, completely locates the primary in space. Once the primary is fixed, the system focus completely locates the secondary (assuming the two mirrors have been made to their nominal radii and conic constants).

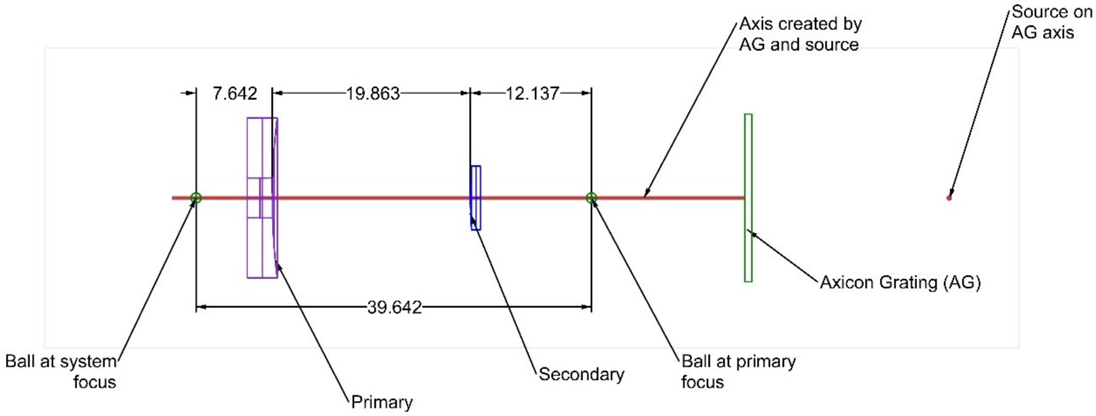


Figure 12. Cassegrain telescope layout with mirror spacings sitting in front of an AG and source on its axis.

The first step in the alignment process is to use the PSM to align balls on posts at the primary and system foci on the axis of the AG. The ball defining the primary focus can be at any convenient location along the axis. The PSM is aligned so the axis spot is centered on the crosshairs and then a ball is centered on the focus of the PSM objective so the return from its center is centered on the crosshairs and in good focus. It is convenient to use a lens mount post to hold the ball. The chamfer around the screw holes in the ends of these posts serve as a good kinematic mount for the balls so they may be removed and replaced and remain well aligned.

The ball at the system focus is installed in the same way but it must be precisely placed to give the 39.642 inch dimension between ball centers. An inside micrometer or measuring rod can be used to set this distance. Figure 13 is a schematic view of how the alignment of the two balls is done.

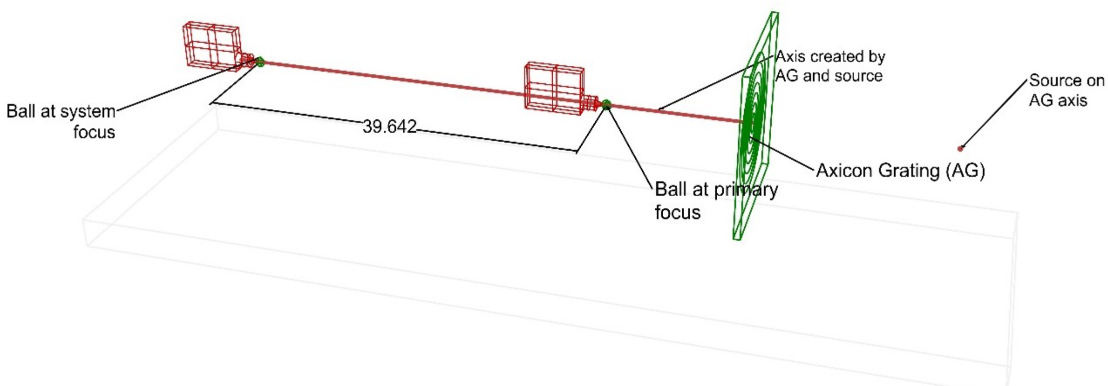


Figure 13. View of how the two balls needed for the alignment are aligned to the AG axis

Once the two balls are aligned, the direction of the PSM is reversed about the prime focus ball and adjusted to focus on the center of the ball so that the PSM is looking toward the primary mirror. Then remove the ball. Insert the primary mirror and adjust it in 3 degrees of translational freedom until the return image is focused and centered on the crosshairs of the PSM. When this condition holds the collimated rays from the parabola are normal to the grating in the paraxial sense. However, unless you were incredibly lucky, the return spot will be comatic indicating that the axis of the parabola is not coincident with the axis of the grating, but rather that the axis of the parabola crosses the axis of the grating at the center of curvature of the parabola.

To finish the alignment of the parabola, rotate it about its center of curvature until the coma is eliminated. In general it will be difficult to rotate the parabola about its center of curvature so it will have to be simultaneously tilted and decentered to accomplish this. The return image must stay centered on the crosshairs to assure the parabola is rotating about its center of curvature and, of course, the PSM is not moved. When the coma is eliminate the axis of the parabola and the AG are coincident. Figure 14 is an illustration of this step in the alignment.

Note in Figure 14 the source for the AG is not shown. This is because once the two balls are aligned to the AG, the axis to which we are aligning is defined by the balls and the AG. At this point even the AG can be removed as long as it is replaced by another flat inserted such that it is parallel with the AG, and the AG can be used in another setup.

Next, focus and center the PSM on the ball at the system focus such that the PSM is looking toward the grating and then remove the ball. Insert the secondary and adjust it in 3 degrees of freedom until the return image is focused and centered on the PSM crosshairs. Again, the image is most likely to be comatic, but by keeping the image on the crosshairs and adjusting the secondary in tilt and decenter the coma can be eliminate and the system will be aligned. This step is shown in Figure 15.

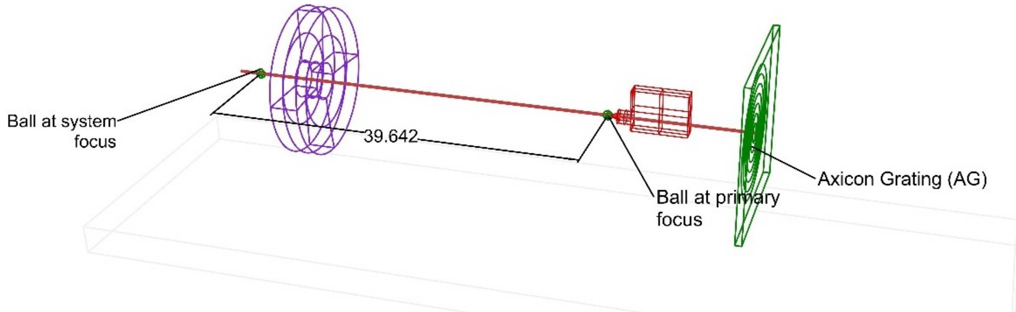


Fig. 14 Alignment of the primary mirror by eliminating coma at the primary focus

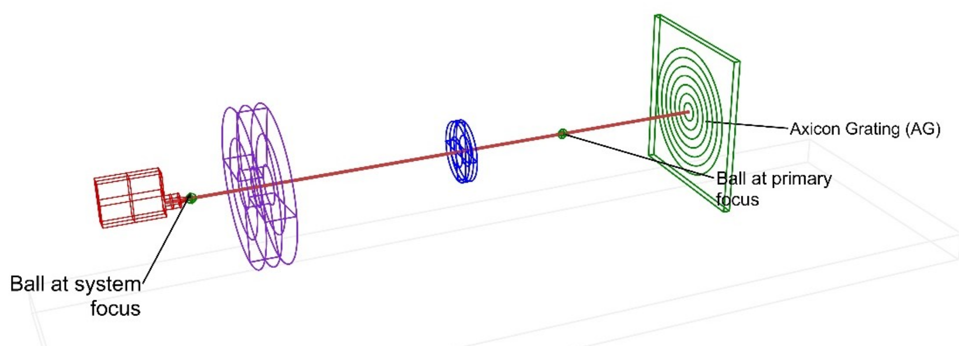


Fig. 15 Alignment of the secondary mirror by eliminating coma at the system focus

Once the alignment fixture with the two balls and a flat situated perpendicular to the axis defined by the balls is initially aligned and used to align a first telescope, the setup may be used repeatedly to align more of the same telescopes. The procedure is identical, first align the primary to its ball and then the secondary (and system) to its ball. The only piece of equipment that is moved is changing the PSM from its position for aligning the parabola to the position for system alignment. If 2 PSMs were used, the one at the system focus could be left there permanently.

10. CONCLUSION

We have shown how to align both refractive and reflective optics using a combination of an autostigmatic microscope and an axicon grating with its own pinhole source. The methods are straightforward and allow lateral alignment of about 1 µm. Angular alignment depends on the length of the optical lever arm but is on the order of seconds of arc or less.

In addition to eliminating the need for a rotary bearing to create an axis in space, the method permits the adjustment of both tilt and decenter of each element without having the move the ASM during that step of alignment. This means the alignment can be accomplished more rapidly because there is good hand/eye coordination without the interruption of having to make another adjustment. Once a particular element is aligned, the alignment setup is re-adjusted axially to the conjugate of the next element to be aligned, adjusted laterally to get the reflected spot centered on the crosshairs and then both the tilt and decenter of that element is aligned in one step. The one caveat is that there must be a means within the cell of adjusting tilt and decenter at each step of the assembly. This has nothing to do with the alignment scheme itself, but with the design of the opto-mechanical components in the first place.

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